Radial Heat Transfer in Packed Beds of Shaped Particles

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Abstract

Experimental data on the effective radial thermal conductivities and wall heat transfer coefficients of cylindrical beds formed of 4-hole and 52-hole cylindrical pellets, 6-spoke wheels and 3-hole trilobed particles are presented. A model with a linear variation of the radial thermal conductivity in the vicinity of the wall is proposed for description of the radial heat transfer in the packed bed. The model allows simple correlation between the wall Nusselt number and the bed core effective radial thermal conductivity. The model does not require any additional empirical parameters for the description of heat transfer in packed beds of different shaped particles.

INTRODUCTION

In order to develop of tubular packed bed heat exchangers, adsorbers, and chemical reactors reliable prediction of heat transfer from the tube wall to the packed bed is very important. A quasi-homogeneous, two-dimensional model is usually used for interpretation of the primary experimental results (temperature distribution in packed beds with fluid flow). Uniformity of flow velocity and radial thermal conductivity in the packed bed is assumed, the axial dispersion of heat is neglected and a boundary condition of the third kind is used at the tube wall [1]. Following many authors let call this model the “Standard Dispersion Model” (SDM). This model has two parameters for description of the radial heat transport: the wall heat transfer coefficient αe and the effective radial thermal conductivity λr eff.

The main disadvantage of the SDM is the prediction of unrealistic fluid temperature near the wall because of the artificial boundary condition of the third kind at the tube wall (the concept of temperature jump at the wall). As a result the reaction rates in packed bed reactors can be predicted incorrectly. To avoid this problem another assumptions on the heat transport mechanism close to the wall are necessary.

The concept based on locally varying radial thermal conductivity λr, r due to the near-wall porosity and velocity changes has gained ground in the last time. Such models are called λr, r-models and used the natural boundary condition of the first kind at the wall (condition of constant wall temperature). For example, Botterill and Denloye [2] separated the packed bed into a core and a wall region of dₚ/2 thickness (dₚ – the diameter of equivalent-volume sphere) with two different radial dispersion coefficients. Cheng and Vortmeyer [3] published a functional relation between radial thermal conductivity and local values of porosity and flow. The thickness of a wall region was assumed to equal to 2.5 dₚ. Tsotsas and Schlunder [4] obtained the distribution function of radial thermal conductivity of the packed bed on the ratio between the thermal conductivity of the particles and that of the fluid. Characteristic thickness of the wall region with varying radial thermal conductivity
can be estimated from calculations of Tsotsas and Schlunder as about 1.5 \( d_p \) then the radial thermal conductivity practically does not depend on the radial position. Winterberg et al. [5] and Winterberg and Tsotsas [6] recommended a two-region model with radial dispersion coefficients depending on the near-wall channeling effects. The convective part of the radial thermal conductivity in the wall region was modelled as a quadratic function of the radial coordinate. Winterberg and co-workers concluded that a constant value of wall region thickness could be used to describe the experimental data with satisfactory results. For packed beds of spheres the thickness of the wall region was found about 0.44 \( d_p \) [5]. In beds packed with cylindrical particles the thickness of the wall region - about 0.4 \( d_p \) [6].

At present work a two-region model with a linear variation of \( \lambda_{er} \) in the vicinity of the wall is proposed for description of heat transfer experiments in packed beds of shaped particles.

\[
\lambda_{er}(r) = \begin{cases} 
\lambda_{er,\text{core}} & \text{for } 0 \leq r \leq R - \delta \\
\lambda_{er,\delta}(r) & \text{for } R - \delta \leq r < R 
\end{cases}
\]  

(1)

In the packed bed core the effective radial thermal conductivity \( \lambda_{er,\text{core}} \) does not depend on radial position \( r \). In the wall region of \( \delta \) thickness \( \lambda_{er,\delta}(r) \) changes linearly from the fluid thermal conductivity \( \lambda_f \) at the wall (\( r = R \)) to the effective radial thermal conductivity \( \lambda_{er,\text{core}} \) in the bed core:

\[
\lambda_{er,\delta}(r) = \lambda_f + \frac{\lambda_{er,\text{core}} - \lambda_f}{\delta} (R - r)
\]  

for \( R - \delta \leq r < R \)  

(2)

This model successfully describes experimental data for beds packed with spheres, cylinders and Rashig rings (Smirnov et al. [7]). One of the virtues of the model is possibility to calculate the thickness of the wall region by one formula for particles of different form. The equivalent hydraulic diameter of the packed bed

\[
d_{eqv} = \frac{4e_{bed}}{d_0 \left(1 - e_{bed}\right)}
\]  

(3)

is used as the characteristic thickness \( \delta \) of the wall region. Here \( e_{bed} \) - bed porosity without account of porosity of grains; \( d_0 \) - the specific surface of one solid particle.

**EXPERIMENTAL**

Measurements of temperature fields in packed beds were performed in steady state experiments without chemical reaction in the tube with inner diameter equal to 84 mm. Test section 650 mm high was cooled from outside by water with flow rate about 25 litres per minute, that guaranteed constant temperature at the inner wall of the tube \( T_w \). Hot air was cooled while passing the tested bed and its temperature was measured in 144 points directly over the layer. Temperature fields were measured at several heights of the layer. Superficial velocities of the air were varied in the range 0.2–2.0 m/s. Bed porosities were measured by a weighting method. Physical properties of the air were considered to be constant in the whole test section.

The effective radial thermal conductivity \( \lambda_{er,\text{core}} \) and the wall heat transfer coefficient \( \alpha_w \) in the packed bed were determined by fitting the SDM-solution to the measured temperature field using the method of least squares. In this work the experimental heat transport parameters were determined using only the temperatures measured in the bed core or at the distance larger than \( \delta = d_{eqv} \) from the wall.

Thereby we assume that the heat transport within the core region is characterised in the same manner as the one-region SDM, the fluid temperature at the tube wall is \( T_w \), and the temperature jump (from boundary condition of the third kind for SDM) takes place within the wall region. Obtained from the SDM value of \( \alpha_w \) defines the heat flux from the packed bed to the wall and can be used as a criterion for experimental verification of a chosen model (see the following chapter).

**CORRELATION BETWEEN \( Nu_w \) AND \( \lambda_{er,\text{core}} \)**

Proposed two-region model allows correlation between experimentally determined SDM-parameters. It results in a simple correlation between the wall Nusselt number and the bed core effective radial thermal conductivity. The wall heat transfer coefficient is defined as:

\[
\alpha_w = \frac{qR}{T_{R=R} - T_w}
\]  

(4)
where $T_{r=R}^{SDM}$ is the gas temperature at the tube wall calculated from the SDM, $q_R$ – the heat flux to the wall. Let us assume that the heat flux in the wall region does not depend on radial position. In this case the temperature jump $T_{r=R}^{SDM} - T_w$ predicted by the SDM can be calculated.

According to the SDM

$$q_R = -\lambda_{er,core} \frac{\partial T}{\partial r}, \quad R - \delta \leq r \leq R$$

(5)

and

$$T_{r=R}^{SDM} - T_{r=R-\delta} = -q_R \frac{\delta}{\lambda_{er,core}}$$

(6)

According to the two-region $\lambda_{er}(r)$-model

$$q_R = -\lambda_{er,\delta}(r) \frac{\partial T}{\partial r}, \quad R - \delta \leq r \leq R$$

(7)

and

$$T_w - T_{r=R-\delta} = -q_R \int_{R-\delta}^R \frac{dr}{\lambda_{er,\delta}(r)}$$

(8)

From eqs. (6) and (8) it follows:

$$T_{r=R}^{SDM} - T_w = q_R \int_{R-\delta}^R \frac{dr}{\lambda_{er,\delta}(r)} - q_R \frac{\delta}{\lambda_{er,core}}$$

(9)

Substituting eq. (9) into eq. (4) we find the relation between $\alpha_w$ and $\lambda_{er,core}$:

$$\alpha_w = \frac{1}{\int_{R-\delta}^R \frac{dr}{\lambda_{er,\delta}(r)} - \frac{\delta}{\lambda_{er,core}}} \frac{\lambda_{er,core}}{\lambda_{er,core}}$$

(10)

The wall Nusselt number (defined through the wall region thickness $\delta$) is:

$$Nu_w = \frac{\alpha_w \delta}{\lambda_f} = \frac{\lambda_{er,core}^*}{\ln \lambda_{er,core}^* - 1}$$

(11)

where $\lambda_{er,core}^* = \lambda_{er,core}/\lambda_f$.

RESULTS

The general correlation for the effective radial thermal conductivity of the packed bed is commonly written in the form:

$$\lambda_{er,core}^* = \lambda_{bed}^* + K \cdot \text{RePr}$$

(12)

where $\lambda_{bed}^* = \lambda_{bed}/\lambda_f$, and $K$ – the convective heat transport parameter.

Table 1 presents the values of $K_{exp}$ which satisfy the eq. (12) for experimental dependences of SDM-parameter $\lambda_{er,core}^*$ on Reynolds number. It was estimated that $\lambda_{bed}^* = 10$ for all studied ceramic particles (Prandtl number was accepted Pr = 0.71 for the air). The accuracy of the experimental determination of $K_{exp}$ was about 15%.

Figures 1 and 2 present $Nu_w$ as a function of Re for shaped particles. Theoretical values of $Nu_w$ were calculated by using eq. (11). In that case $\lambda_{er,core}^*$ was calculated from eq. (12) using determined values of $\lambda_{bed}^*$ and $K_{exp}$. Experimental points of $Nu_w$ were calculated from values of SDM-parameter $\alpha_w$. The Reynolds number is based on the equivalent particle diameter $d_p$ (the hole volume was included in the particle volume). The obtained correlation (11) for the wall Nusselt number is in a good agreement with the experimental data.

TABLE 1

<table>
<thead>
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<th>No.</th>
<th>Grain</th>
<th>$\varepsilon_{bed}$</th>
<th>$K_{exp}$</th>
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| 1   | Ceramic 4-hole pellet: outer diameter 14 mm, length 17 mm,  
hole diameter 4 mm | 0.40 | 0.20 |
| 2   | Ceramic 52-hole pellet: outer diameter 19 mm, length 17 mm,  
square holes 1.5x1.5 mm | 0.48 | 0.14 |
| 3   | Ceramic 6-spoke wheel: outer diameter 18 mm, length 16 mm,  
wall thickness 2 mm | 0.42 | 0.17 |
| 4   | Ceramic 3-hole trilobed particle: outer diameter of each lobe 7.5 mm,  
length 11 mm, hole diameter 3.5 mm | 0.38 | 0.24 |
Fig. 1. Dependences of wall Nusselt number on Reynolds number for beds of 4-hole and 52-hole cylindrical pellets. Open circles – ceramic 4-hole pellets, diameter 14 mm, length 17 mm, holes diameter 4 mm, open squares – ceramic 52-hole pellets, diameter 19 mm, length 17 mm, holes 1.5x1.5 mm; lines – eq. (11).

Fig. 2. Dependences of wall Nusselt number on Reynolds number for beds of 6-spoke wheels and 3-hole trilobed particles. Open circles – ceramic 6-spoke wheels, diameter 18 mm, length 16 mm, wall thickness 2 mm, open triangles – ceramic 3-hole trilobed particles, diameter of each lobe 7.5 mm, length 11 mm, hole diameter 3.5 mm; lines – eq. (11).

CONCLUSION

The two-region $\lambda_{er}(r)$-model was proposed for description of the radial heat transfer in the packed bed. The model allows simple correlation between the wall Nusselt number and the bed core effective radial thermal conductivity. The obtained correlation stands comparison with the experimental data.

The model with a linear variation of $\lambda_{er}$ in the vicinity of the wall does not require any additional empirical parameters for the description of heat transfer in packed beds of shaped particles. The characteristic thickness of the wall region $\delta = d_{eq}$ is defined by means of the bed porosity and the specific surface of the single particle. Thus the model with a linear variation of $\lambda_{er}$ in the vicinity of the wall was verified for beds of 4-hole and 52-hole cylindrical pellets, 6-spoke wheels and 3-hole trilobed particles.

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REFERENCES