Structure of the upper crust beneath volcanoes of the Klyuchevskoy group revealed from ambient noise tomography

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Supplementary materials

DATA

A large seismic network of temporary seismic stations named KISS was deployed by an international consortium of researchers from Russia, France and Germany [Shapiro et al., 2017b] in northern Kamchatka in the period from August 2015 to July 2016. It covered an area of 120x70 km and included the Klyuchevskoy group of volcanoes with the surrounding part of the Central Kamchatka depression and reached the Kizimen volcano in the south. In total, the KISS network contained 83 temporary seismic stations in addition to 23 permanent stations of the KF FRC EGS RAS. The temporary network included 30 Trillium Compact sensors (with a maximum period of up to 120 s), six Guralp CMG-6T sensors (30 s), eight Guralp CMG-6TD sensors combined with data loggers (30 s), nine R-Sensors CME-4111 and 30 Mark L-4C-3D with a nominal maximum period of 1 s. It should be noted that despite the specified frequency range, the Mark L-4C-3D sensors stably registered a signal with a period of up to 20 seconds, which allowed us to use them for performing ambient noise tomography. Thus, the entire seismic network consisted of 50 broadband stations, 27 stations with intermediate frequency characteristics and 28 short-period instruments.

The dispersion curves of the group velocities of Rayleigh waves were estimated from the calculated correlograms using the procedure of frequency-time analysis (FTAN) [Levshin et al., 1989; Ritzwoller and Levshin 1998], which was implemented as GUI_FTA_v2p1 software [Mordret A., Landès M., 2013]. At the first stage, we averaged the positive and negative parts of the correlogram, which corresponded to the waves travelling from one station to another and back. After that, the correlograms were filtered with a series of narrow-band filters [Shapiro and Singh, 1999] and were used to assess the systematic error. The filtered correlograms were plotted in a single plot as a function of apparent velocity versus period. The maximum amplitude of the filtered correlogram at each period corresponded to the value of the group velocity at a given frequency. For all pairs of stations, such dependences were plotted and used to derive the dispersion curves following the maximum amplitudes in the filtered correlograms. For some pairs, due to the low signal-to-noise level, it was impossible to unambiguously determine the maximum points, and such data were discarded. Even where the dispersion curve was relatively well traced, at small
periods, determining the position of the curve was hampered by the presence of several modes, and at high periods, by the too flat shape of the spectrum. For these reasons, the majority of dispersion curves was taken for a range of periods of 2-7 seconds.

AMBIENT NOISE TOMOGRAPHY ALGORITHM AND RESULTS

In the present study, the tomographic inversion was performed using the algorithm described in [Koulakov et al., 2016], with some modifications. The procedure consisted of two stages: the construction of two-dimensional maps of group velocities for selected frequencies and the inversion for a three-dimensional distribution of the shear wave velocity (Vs).

**Group velocity maps.**

The group velocity maps were constructed based on iterative recurrences of the ray tracing and the inversion procedures. For each selected frequency (period) for all pairs of stations, the rays of surface waves were traced in 2D taking into account the relief and distribution of the group velocity. For this purpose, a special bending ray tracing algorithm has been developed, which is described in detail in [Koulakov et al., 2017]. This algorithm performs a gradual bending of the ray paths and searches for a trajectory with a minimum travel time. Examples of ray construction for the first and fifth iterations for one of the frequencies are shown in Fig. 1a, b. Note that even in the starting model with the constant group velocity, the rays are not straight due to the influence of the relief (see, for example, the ray marked in blue in Fig. 1a, b). In the presence of high sharp mountains, as in our case, some surface waves has smaller travel times if they travel around the mountain and not along its surface.

The velocity model was parameterized using a set of nodes installed in areas where the ray density exceeds a certain threshold (in our case, 10%) of the average value. In such areas, the nodes are distributed regularly with the spacing of 2 km (Fig. 1c). Between the nodes, the velocity is approximated using the bilinear interpolation. To minimize the effect of the parameterization grid geometry on the result, a series of independent inversions were performed based on the grids with different basic orientations (0, 22, 45, and 66 degrees). Then the results were averaged and combined into one model.

The starting model for calculating the residuals in the first iteration was a constant velocity, which was determined for each frequency so that the summary time residuals along all rays were equal to zero. In this case, if the maximum residual exceeded a predefined threshold (2 seconds in our case), then this ray was discarded, and the new average velocity was recalculated. The procedure was repeated until all residuals became below the threshold.

The inversion was carried out by solving a system of linear equations using the LSQR method [Paige and Saunders, 1982; Nolet, 1987]. Smoothing of the model and the amplitude of
anomalies were controlled by additional equations that allowed minimizing either the difference in the values of the anomalies in adjacent nodes or the amplitude of the solution at each node.

After performing the inversion, the rays were traced in the new 2D model. Then the new residuals and the sensitivity matrix were calculated and inverted. In total, we performed five iterations. The resulting group velocities for different periods are presented in the main part of this article. Significant differences in the structures of the anomalies are observed depending on the period. This shows that the velocity structure in the study region changes significantly both in the lateral direction and in depth. An interpretation of this model in terms of geology is presented in the “Discussion” section.

**Optimization of the one-dimensional Vs model.**

The two-dimensional group velocity models derived in the previous step were used as input data to construct the three-dimensional model of the shear wave velocity (Vs). Before implementing the three-dimensional inversion, the reference one-dimensional model of Vs (z) was optimized based on the average values of the group velocities at each frequency. Optimization of a one-dimensional model is performed using an iterative inversion procedure. For a given starting velocity model Vs (z), the dispersion curve is calculated using a forward modeling algorithm developed by Herrmann [1987]. The data vector for inversion is calculated as a difference between the calculated and observed dispersion curves. For the selected reference model, by applying the calculations according to the Herrmann scheme [1987], we calculated the sensitivity matrix Aij, which indicates how the velocity variation at the i-th level dVs_i affects the change in the group velocity at the j-th frequency, dU_j.

\[ A_{ij} = \frac{dU_j}{dVs_i} \]

A graphical representation of this matrix in the form of sensitivity kernels is shown in Fig. 2. It can be seen, for example, that at the first iteration for a wave with a period of 2 s, the maximum sensitivity is achieved at a depth of 0.5 km, for 3 s - 1.5 km, for 4 s - 2 km, for 5 s - 3 km; for 6 s - 4 km and for 7 s - 6 km. The maximum depth, at which low frequency waves are still sensitive to the Vs changes, is estimated at about 8 km.

The system of linear equations is solved using the LSQR algorithm [Paige and Saunders, 1982; Nolet, 1987] with the inclusion of additional blocks for the model smoothing and amplitude damping. In our case, the smoothing and amplitude dumping coefficients were 5 and 5, respectively. The anomalies obtained after the inversion were added to the reference model (thin black lines in Fig. 3), in which a new modelled dispersion curve and new values of the sensitivity matrix were calculated. After that, an inversion was performed again, and a new model was calculated. In our case, a total of five iterations were performed. The final velocity model and the corresponding dispersion curve are shown in Fig. 3 with a red line. As can be seen, the obtained
dispersion curve does not perfectly fit the observed curve. We tried different values of the smoothing parameters and found that with the use of smaller damping (for example, 1), the reconstruction of the dispersion curve was almost perfect. However, the resulting velocity model had sharp variations in velocity that would lead to unrealistic structures in the final model. Therefore, we have chosen a more conservative solution for the reference velocity distribution based on a stronger damping.

An important step is to check the vertical resolution provided by the existing observations. Initially, we did not expect high vertical resolution, since we used dispersion curves for only six periods with a relatively small range. This does not provide enough equations to reconstruct detailed structures in depth. Figure 4 shows an example of synthetic modeling aimed at assessing the vertical resolution in this case for a 1D velocity model. We have chosen the distribution obtained for real data as a reference model. On this velocity model, we superimposed layers 2 km thick with alternating positive and negative anomalies with an amplitude of ± 7%. In this model, the dispersion curve was calculated (blue line in Fig. 4c). It can be seen that the dispersion curve in the starting model was significantly different from the curve in the true model. After several iterations, the procedure yielded a solution shown by the red line in Fig. 4c, which matches the “observed” values quite accurately. In this case, the initial and reconstructed velocities and anomalies are shown in (Fig. 4a and b) with the corresponding colors. As expected, the ideal reconstruction of the layers was not achieved. However, the main variations to a depth of 5-6 km were determined correctly. The limitations of vertical resolution demonstrated by this test should be taken into account when interpreting the results of the inversion of real data.

**Calculation of the three-dimensional Vs models.**

To construct a three-dimensional model Vs at each point of the region (x, y), a local dispersion curve was constructed from the values of group velocities in two-dimensional models corresponding to different frequencies. The optimization algorithm described in the previous section allows constructing a one-dimensional model Vs(z) at the current point of the region. By performing a similar procedure for all points of the region, we build a three-dimensional model Vs(x, y, z). In this work, the inversions for all points of the region were carried out simultaneously. In this case, additional conditions were imposed aimed at smoothing the velocity in the vertical and horizontal directions. To do this, we added equations responsible for minimization of the velocity differences in neighboring nodes located at the same depth. In the present study, the smoothing coefficients in the vertical and horizontal directions were 0.2 and 0.3, respectively. The amplitude damping was not performed in this case.

After calculating the three-dimensional velocity model, the residuals between the model and observed group velocities at each point were calculated and a new sensitivity matrix was
constructed taking into account the updated Vs(z) values at each point of the region (x, y). Then the inversion was carried out again and a new velocity model was built. To calculate the final model, we performed five iterations.

The horizontal and vertical sections of the 3D model obtained as a result of experimental data inversion are given in the main part of the article. It should be noted that horizontal sections are not, in the strict sense, such, since the depth for them is measured from the day surface and follows the relief. Moreover, the greater the depth of the section is, the smoother the reference relief appears for it. This is due to the fact that the information about greater depths is provided by lower frequency surface waves, which are less sensitive to the topography variations. Earlier it was noted that when optimizing Vs (z), we decided to use a smoother velocity model, which did not provide a perfect match with the dispersion curve. In this regard, during inversion at some depths, the velocity anomalies turned out to be not perfectly balanced and biased to the positive or negative side. Therefore, for each depth, we present a separate color scale, in which the transition from “blue” to “red” does not always coincide with zero anomaly. For all sections, the results are shown only where the summary sensitivity is at least 1% of the average summary sensitivity over the entire region. Thus, in places where there are no data, or the ray density is too low, the results are not displayed.

When comparing the maps of the group velocities of Rayleigh waves and horizontal sections of the final three-dimensional model Vs, one can see that group velocities at small periods correspond to shallow depths, and large periods correspond to large depths. For example, the structure at a depth of 1 km is coherent with the group velocity at a period of 2 s, at a depth of 2 km by a period of 3 s, and at a depth of 6 km by a period of 7 s. An exception is the depth of 4 km, where the influences of group velocities at periods from 5 s to 7 s is mixed.

In the main article, we present four vertical sections with S-wave velocity anomalies and absolute velocities. As mentioned above, the depths in the resulting velocity model are considered relative to the relief. For this reason, at shallow depths, layers with positive and negative anomalies follow the relief lines, although, in fact, these layers correspond to a constant depth in the resulting 3D model. At great depths, the influence of the relief gradually decreases in such a way that at the lower boundary of the sections (8 km b.s.l.), the velocities are computed with respect to the flat topography. The interpretation of the obtained three-dimensional Vs model is given in the section "Discussion of the results".

**Synthetic testing.**

To check the spatial resolution of the obtained models and find the optimal values of the inversion parameters, a series of synthetic tests was carried out. Each test was performed according to the following scheme:
1. A synthetic S-wave velocity model (Vs) is defined as a sum of a one-dimensional reference model and a three-dimensional distribution of synthetic anomalies.

2. For a given synthetic velocity model, the group velocity maps are calculated for all frequencies using the forward modeling algorithm [Herrmann, 1987].

3. The rays corresponding to the pairs of stations in the experimental dataset are traced in the derived 2D group velocity models for each frequency. This step uses the bending algorithm, same as used for the iterative inversion of experimental data. Travel times are calculated for these rays and converted to apparent group velocities along each ray.

4. The file with apparent group velocities for all rays and frequency is the input data for the following recovery of the synthetic model. At this step, we “forget” about the synthetic velocities and start performing the same procedure as in the case of the experimental data inversion.

5. Based on these synthetic data, we calculate a set of the 2D group velocity maps for the given frequencies.

6. The resulting group velocity maps are inverted to the three-dimensional velocity distribution Vs. Then the resulting velocity distribution is compared with the original “true” synthetic model. If the structures are similar, the resolution appears to be good.

Fig. 5 shows the recovery result of the checkerboard synthetic model in which the synthetic anomalies are defined as alternating squared patterns. In this case, they had a lateral size of 15x15 km and an amplitude of ± 9%. Since the purpose of this test was to study horizontal resolution, the anomaly values were set unchanged with depth. It can be seen that our data configuration allows us to reliably reconstruct the lateral shape of anomalies down to the depths of ~6 km. At the same time, the amplitude of anomalies in deep sections is significantly lower than that in the true synthetic model, which is caused by a decrease of the surface wave sensitivity with depth.

As was noted in section "Optimization of the one-dimensional Vs model", the vertical resolution in this case has certain limitations, mainly due to a relatively small number of different periods used in inversion. Figure 6 shows the result of synthetic testing aimed at studying the vertical resolution. In this case, a synthetic model of the "checkerboard" type with velocity anomalies of 15x2 km in size was defined along section A1-B1, the position of which is indicated in (Fig. 5). It can be seen that the tomographic inversion provides a fair recovery of anomalies in the upper three layers down to a depth of about 5 km. In the fourth layer, the anomalies are smeared downward, which indicates a limited resolution of the model below a depth of 6 km.

The resolution limitations revealed by the results of synthetic testing should be taken into account when interpreting the results of inversion of experimental data.
**Fig. 1.** a. Example of the ray paths construction at the first iteration of the inversion procedure for the period of 5 s. The gray and blue lines represent the ray paths, the red triangles are the seismic stations. The ray paths highlighted in blue illustrate the deviation of the path from a straight line due to the influence of the relief. b. Example of the ray paths and group velocity map construction at the fifth iteration of the inversion procedure for the period of 5 s. The stations and the rays are indicated as in the previous figure. c. Example of the parameterization grid construction for data corresponding to the period of 5 s. The colors indicate the ray density, the black dots are the parameterization nodes. The gray lines represent the links between the nodes that are used to smooth the model. The relief is indicated by contour lines with intervals of 500 m between them.
Fig. 2. Sensitivity kernels of the Rayleigh wave group velocities for different periods at the first, second and fifth iterations of the inversion procedure.
Fig. 3. Search for the optimal average velocity model $V_s(z)$ for experimental data. 

**a.** Velocity distributions with depth. The blue line shows the starting model, the black lines are the intermediate models and the red line is corresponding to the final model obtained after 5 iterations. 

**b.** Dispersion curves corresponding to the models in (fig.3a). The blue line indicates the observed dispersion curve, the dispersion curves corresponding to the intermediate models are shown with the black lines and the red line is the dispersion curve calculated for the final model.
Fig. 4. Synthetic test for examining vertical resolution in a one-dimensional model of $V_s(z)$. a. Absolute velocities of one-dimensional models. b. Relative velocity anomalies of one-dimensional models. The green lines correspond to the starting velocity model, the blue lines correspond to the true model, the black lines correspond to the intermediate results and the final model recovered after 8 iterations is shown in the red lines. c. Dispersion curves corresponding to the starting (green line), true (blue line), intermediate (black lines) and final (red line) models.
Fig. 5. Results of the checkerboard test on horizontal sections. The amplitude of the anomalies is constant with depth. The depth of each section is indicated in the lower left corner. The gray lines show the configuration of the synthetic anomalies. The relief is indicated by contour lines with intervals of 500 m between them. The map corresponding to the horizontal section at the depth of 4 km shows the location of the vertical section presented in (fig. 6).
Fig. 6. Result of the vertical checkerboard test for the section with the location shown in (fig. 5). The gray lines show the configuration of the synthetic anomalies. The vertical scale is twice the horizontal scale.